



Problem of the Month

Perfect Pair



Level A

When you add two numbers you get a certain answer. Using the same two numbers, subtract the smaller from the larger number. If the two answers are the same, we will call that a perfect pair.

Can you find two numbers that are a perfect pair?

If you think it is impossible, explain why.

If you have found a perfect pair, explain why they are a perfect pair.

If it is possible, is there more than one perfect pair? How many are there?

Explain all you know about perfect pairs and what is special about them.

Level B

Each shoe represents a digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). The style of shoe always represents the same digit in all of the number sentences. Examine the number sentences.

$$\text{Pink High Heel} + \text{Pink High Heel} + \text{Pink High Heel} = \text{Brown Slipper}$$

$$\text{Blue Sneaker} - \text{Brown Cowboy Boot} = \text{Blue Sneaker}$$

$$\text{Brown Sneaker} \times \text{Brown Sneaker} = \text{Brown Sandal}$$

$$\text{Black Heel} - \text{Brown Sneaker} = \text{Orange Sneaker}$$

$$\text{Grey Boot} = \text{Grey Boot} \times \text{Blue Sneaker}$$

$$\text{Brown Cowboy Boot} = \text{Black Heel} \times \text{Brown Cowboy Boot}$$

$$\text{Brown Sandal} = \text{Brown Sneaker} + \text{Brown Sneaker}$$

$$\text{Brown Slipper} - \text{Pink High Heel} = \text{Grey Boot}$$

$$\text{Blue Sneaker} - \text{Brown Sandal} = \text{Brown Sandal}$$

Pair each shoe with a digit. Explain your reasoning.



Level C

A perfect trio involves three whole numbers. Using the three numbers, add the first two numbers together then divide the sum by the third number. Using the same three numbers, subtract the second from the first number and then multiply the difference by the third. The trio is perfect if the two outcomes are equal.

Can you find three whole numbers that are perfect trios?

If not, how can you show that there are not any perfect trios? Is there a way to prove there are no perfect trios? Explain completely.

If you can find a perfect trio, is there more than one perfect trio? If so, how many? List the trio(s) you found. How do you know if you found them all? Describe any special characteristics of perfect trios. How can you go about finding them? Explain completely.

Level D

In this problem, a perfect pair is defined as two numbers whose sum is equal to their product.

Explore these perfect pairs.

If you cannot find any perfect pairs, prove that a perfect pair cannot exist.

If you find perfect pairs, then generalize your findings and describe the relationship of the number pairs. Illustrate the set of perfect pairs using multiple representations (words, symbols, graphs, tables, diagrams).

Level E

In this problem, a perfect pair is defined as two numbers whose sum is equal to their quotient.

Explore these perfect pairs.

If you cannot find any perfect pairs, prove that a perfect pair cannot exist.

If you find perfect pairs, then generalize your findings and describe the relationship of the number pairs. Illustrate the set of perfect pairs using multiple representations (words, symbols, graphs, tables, diagrams).