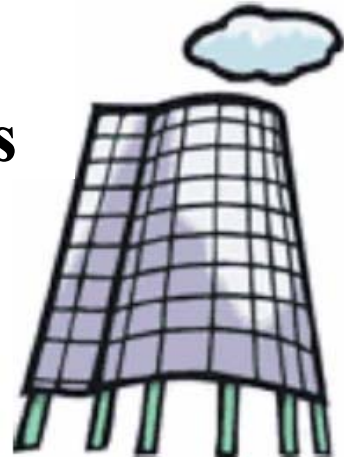


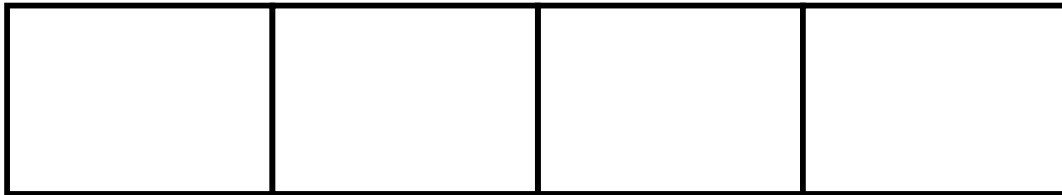
Problem of the Month

Infinite Windows



Level A

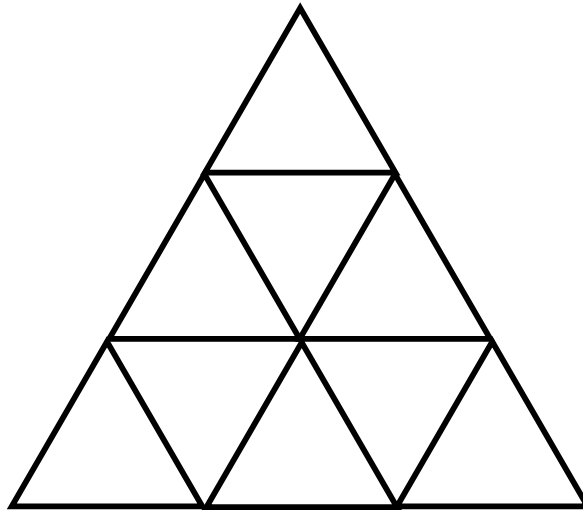
Tracy and Debbie are counting rectangles. The figure below is made of different rectangles and rectangles of different sizes.



How many rectangles can you find in the figure above? Show and explain how you found all of them.

Level B

Ramis and Aaron are counting triangles. The figure below is made of several equilateral triangles of different sizes.



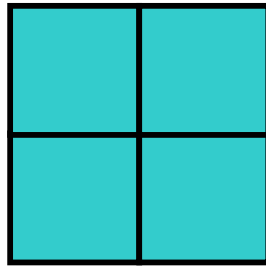
How many triangles can be found in the figure above? Explain your findings.

How do the sizes of the triangles compare? How do they compare in terms of area? How do they compare in terms of length?

If one more row were added to the bottom of the figure, how many triangles would be found in the new figure?

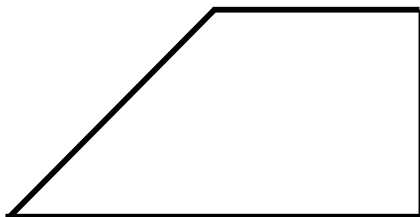
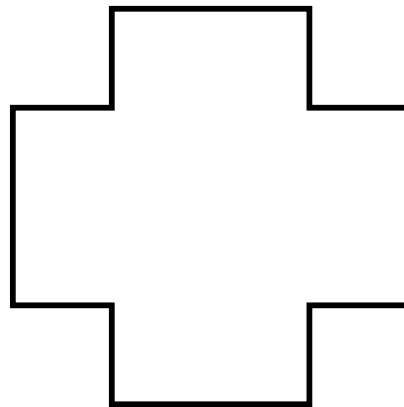
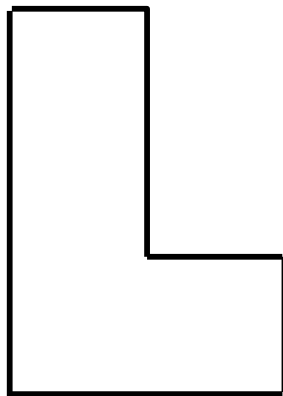
Level C

A *Perfect Polygon Puzzle* is a set of congruent polygon pieces that form to make a larger polygon similar to the puzzle pieces. For example, a square can be a perfect polygon puzzle since four squares can be arranged to make a larger square, as shown below:



Do all regular polygons have this property? Explain your answer fully.

Other non-regular polygons can be *perfect polygon puzzles*. Which of the figures below are *perfect polygon puzzles*. If they are, show how they are. If they are not, explain why.



Level D

Have you ever sat in a hair salon or barbershop and looked through opposite mirrors? You can look down an endless tunnel of images that are all the same. What you see is the front of your head and the back of your head in a series of identical pictures, appearing smaller and smaller, forever. Likewise, if you look at a picture of a girl holding a mirror while looking in a mirror, you experience the same thing. Examine the picture of the girl with the mirror below:



How do the heights of the girl in the smaller pictures compare to the height of the girl in the original picture?

If you had a microscope and could zoom in on the tenth picture of the girl, what would be the actual height of the girl in that picture? Explain.

Suppose you could use a microscope to see something 0.0001 millimeter tall. How many pictures of the girl would you be able to see then? Explain.

Level E

Sierpinski's Carpet is a fractal. It is created in stages. The initial stage (Stage 0) is a square. Stage 1 has a square hole cut out of the center of the original square exactly one ninth of the area of the original square.

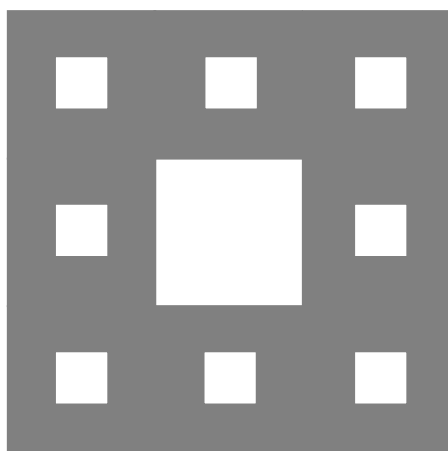


Stage 0



Stage 1

Stage 2 is created from Stage 1. It has eight new smaller square holes cut out of the remaining area. Where do those holes appear to be on the original square and how big are the square holes? State the area of the small hole in terms of the fractional part of the original square (Stage 0). If the original square was 1 square unit in area, what fractional part of the area of the square remains in Stage 2? Explain.



Stage 2

- What do the next three stages of the fractal look like (draw or describe)?
- At Stage 5, what area of the original square remains?
- If the perimeter of the fractal includes all the lengths of borders of the square, including the outside, as well as the length that surrounds each hole, what would be the perimeter of each stage (0 to 5) considering the length of the side of Stage 0 as one unit in length?

- Since all fractals have an infinite number of stages, what do you think the area approaches and the perimeter approaches as you get larger and larger stage numbers? Explain.